Expressiveness vs Complexity: the Case of a Logic for Reasoning about Time

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1. Motivations

Formal logic is a breathtakingly versatile tool used in Artificial Intelligence (e.g., Knowledge Representation), mathematics, cognitive science, philosophy, among others, where by logic we mean a system which given premisses enables to infer their consequences. Two main properties of such systems are:

- *Expressive power* – what can be expressed in the language of a logic.
- *Computational complexity* – how much time/memory is needed to perform reasoning in a logic.

In general, **better expressiveness** has the price of **worse complexity**:

← Better Expressive Power

Better Computational Complexity →

As a result, the *big questions* are:

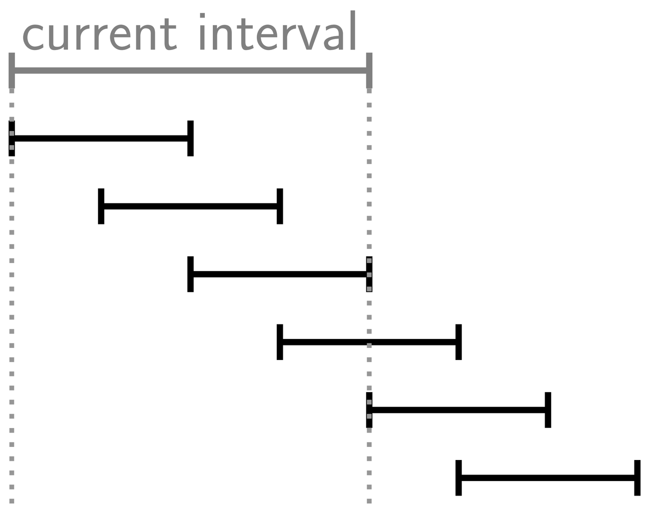
- ▶ What reasoning may be performed (which logics are decidable)?
- ▶ What reasoning may be performed efficiently (which logics are in the complexity class P)?

My research refers these questions to the case of *Halpern-Shoham logic*.

2. Halpern-Shoham Logic (HS)

HS is a modal logic for *reasoning about temporal intervals*. Its modal operators enable us to access an interval which:

- *begins* the current interval (B)
- proceeds *during* the current interval (D)
- *ends* the current interval (E)
- *overlaps* the current interval (O)
- *is adjacent to* the current interval (A)
- *is later than* the current interval (L)



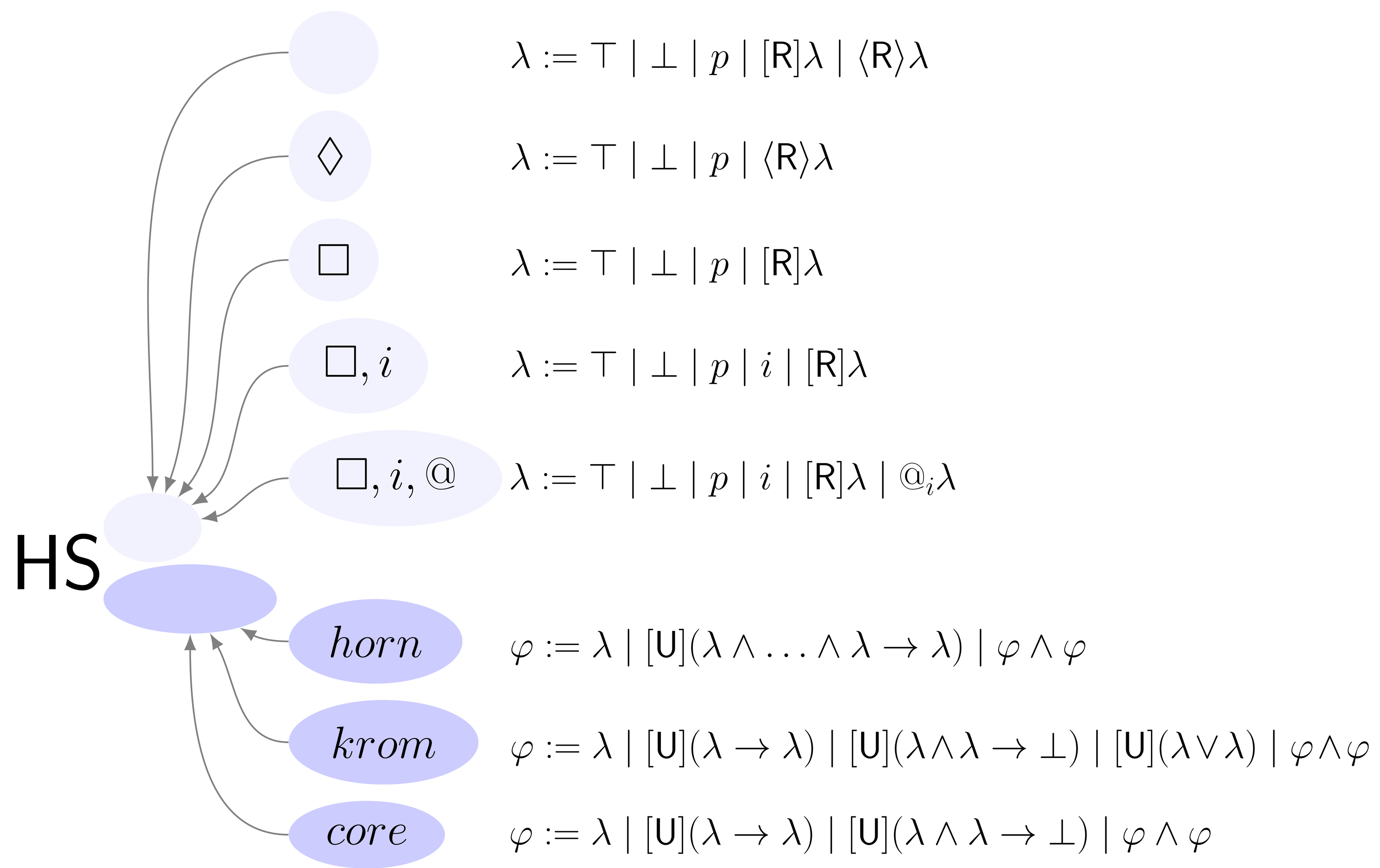
or is in an inverse relation: $\bar{B}$, $\bar{D}$, $\bar{E}$, $\bar{O}$, $\bar{A}$, $\bar{L}$. Hence, the modal operators are:

$\langle R\rangle\psi$ – ψ holds *in some* interval that is in relation R with the current one;
$[R]\psi$ – ψ holds *in all* intervals that are in relation R with the current one;
where ψ is a formula, and $R \in \{B, \bar{B}, D, \bar{D}, E, \bar{E}, O, \bar{O}, A, \bar{A}, L, \bar{L}\}$.

Example: the formula $[L][\bar{L}](conference \to \langle B\rangle opening \wedge \langle E\rangle closing)$ states that each conference is begun by an opening and ended by a closing.

3. HS Fragments

Full HS is *undecidable*, so we search for its better-behaved fragments. Let:



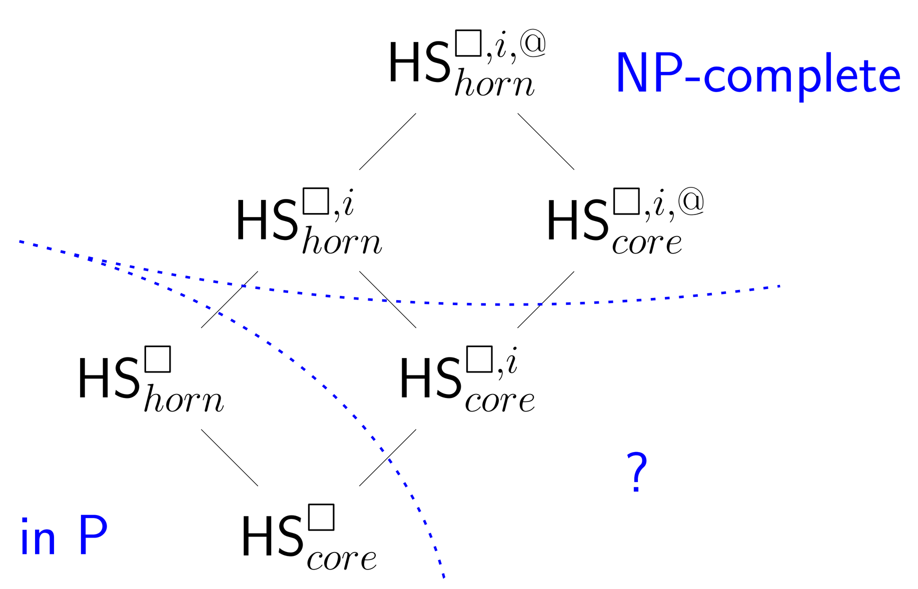
Where:
- p is a propositional variable;
- i is a nominal (variable which is true in exactly one interval);
- $@_i\psi$ states that ψ holds in interval i;
- $[U]\psi$ states that ψ holds in all intervals.

To obtain decidability we need to *disallow discrete and irreflexive time lines*.

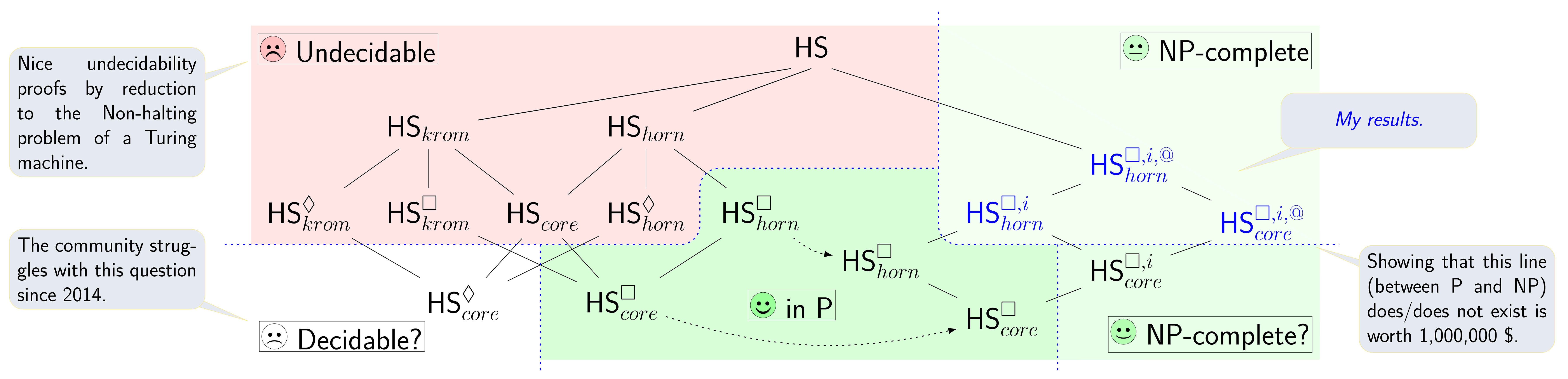
▶ *Particularly interesting is* $HS^{\Box}_{horn}$ which is expressive enough for applications to temporal databases and computationally efficient (in class P).

4. My Results

1. *Augmenting* $HS^{\Box}_{horn}$ *and* $HS^{\Box}_{core}$ *with the ability of referring* to single temporal intervals *does not lead to undecidability*.
2. *The price for such referentiality* is (in most cases) *NP-completeness* (i.e., probably a loss of the efficiency of reasoning).



5. Overall Results – the Complexity Map of HS Fragments



6. Conclusions and Future Work

I have *constructed referential extensions of* $HS^{\Box}_{horn}$ *and* $HS^{\Box}_{core}$, and *proved their NP-completeness* (except $HS^{\Box,i}_{core}$), hence *decidability*.

Interesting *open questions* include the following:

- ▶ What is the complexity of $HS^{\Box,i}_{core}$ (when discrete and irreflexive time lines are disallowed)?
- ▶ Is $HS^{\Diamond}_{core}$ decidable?
- ▶ Which HS fragments allow referentiality without i and $@_i$?

7. References

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