Reasoning with movement based on qualitative representation

Przemysław Wałęga

2014,

Outline

Propositional Dynamic Logic

PDL language

- $\mathbb{V}$ – propositional variables,
- $\mathbb{RC}$ – relational constants (atomic programs),
- $\{\cup, ;, ?, ^*\}$ relational operations
  - $\cup$ nondeterministic choice,
  - $;$ sequential composition,
  - * iteration,
  - $?$ test performance,
- $\{\neg, \wedge, \vee, \rightarrow, [], \langle\rangle, \mathbf{0}\}$ propositional operators,

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Intuitive meaning

- $[\alpha]\varphi$ – "It is necessary that φ after executing α"
- $\alpha \cup \beta$ – "Choose either α or β nondeterministically and execute it"
- $\alpha; \beta$ – "Execute α, then execute β "
- α^* – "Execute α a nondeterministically chosen finite number of times"
- $\varphi?$ – "Test φ proceed if true, fail if false"

Remark

$\langle\rangle$ is definable by $[]$:

- $\langle\alpha\rangle\varphi \stackrel{\text{df}}{=} \neg[\alpha]\neg\varphi$

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for formulas φ , ψ and set of atomic formulas Φ_0
programs α , β and set of atomic programs Π_0
set of formulas Φ and set of programs Π are the smallest sets such that:

- $\Phi_0 \subseteq \Phi$
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- if $\varphi, \psi \in \Phi$, then $\neg\varphi, \varphi \wedge \psi, \varphi \vee \psi, \varphi \rightarrow \psi \in \Phi$
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Set of formulas Φ and set of programs Π
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Semantics

Kripke Model

$$\mathfrak{K} = (K, m_{\mathfrak{K}})$$

- K – not-empty set of states: k,u,v, ...
- $m_{\mathfrak{K}}$ – meaning function:
  - $m_{\mathfrak{K}}(\varphi) \subseteq K, \qquad \varphi \in \Phi$
  - $m_{\mathfrak{K}}(\alpha) \subseteq K \times K, \quad \alpha \in \Pi$

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Model example $\mathfrak{K} = (K, m_{\mathfrak{K}})$

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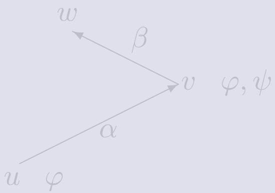

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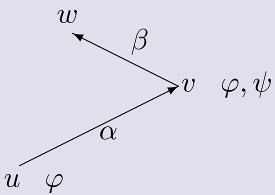

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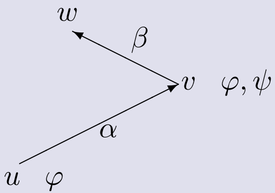

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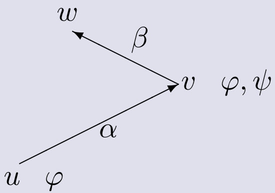

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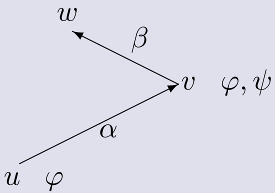

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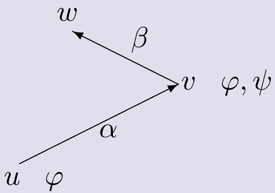

Propositional Dynamic Logic

Programs construction

$$\textbf{skip} \stackrel{df}{=} 1?$$
$$\textbf{fail} \stackrel{df}{=} 0?$$
$$\textbf{if } \varphi \textbf{ than } \alpha \textbf{ else } \beta \stackrel{df}{=} (\varphi?;\alpha) \cup (\neg\varphi?;\beta)$$
$$\textbf{while } \varphi \textbf{ do } \alpha \stackrel{df}{=} (\varphi?;\alpha)^*;\neg\varphi?$$
$$\textbf{repeat } \alpha \textbf{ until } \varphi \stackrel{df}{=} \alpha;(\neg\varphi?;\alpha)^*;\varphi?$$

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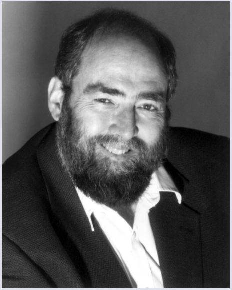
Qualitative approach



,,People who have never heard of differential equations successfully reason about the common sense world of quantities, motion, space, and time."

Kenneth D. Forbus

Qualitative approach

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- qualitative representation,
- qualitative reasoning.

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Logic

Qualitative reasoning method PDL_M^F:

MUNOZ-VELASCO, E,; BURRIEZA, A.; OJEDA-ACIEGO, M.:
A logic framework for reasoning with movement based on fuzzy qualitative representation.
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Idea

Movement representation

Movement is represented by a tuple L:

$$L = L_1 \times L_2 \times L_3 \times L_4 \times L_5 \times L_6 \times L_7$$

- $L_1 = A \times A$, for $A = \{A_1, \ldots, A_k\}$, $k \in \mathcal{N}$,
- $L_2 = 2^{\{v_0, v_1, v_2, v_3\}} \setminus \varnothing$,
- $L_3 = 2^{\{o_0, o_1, o_2, o_3, o_4\}} \setminus \varnothing$,
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Example

$$(A_i, A_j; v_2v_3; o_3; +, -; o_1o_2o_3; o_1, d_1d_2; o_3, d_0)$$

Idea

Composition tables

A_iA_j \ A_jA_k	$*0$	$*-$	$*+$
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Syntax

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- $\Phi_0 = \mathbb{V} \cup L$,
  - $\mathbb{V}$ – set of propositional variables,
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Model $\mathcal{M}$

$$\mathcal{M} = (W, m)$$

- W – not empty set of states denoted by tuples L,
- m – meaning function:
  - $m(\varphi) \subseteq W$
  - $m(\alpha) \subseteq W \times W$

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PDL_M^F logic is:

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Application

Scenario

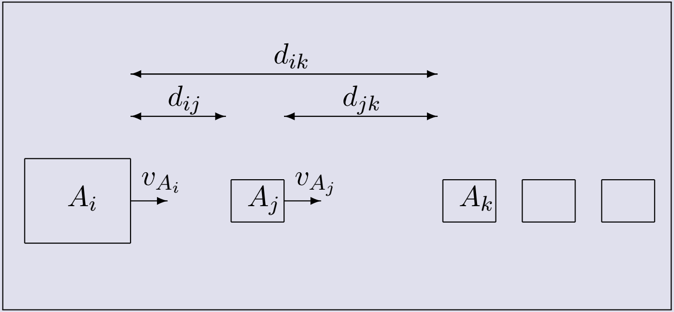

Figure : The scenario setup.

Application

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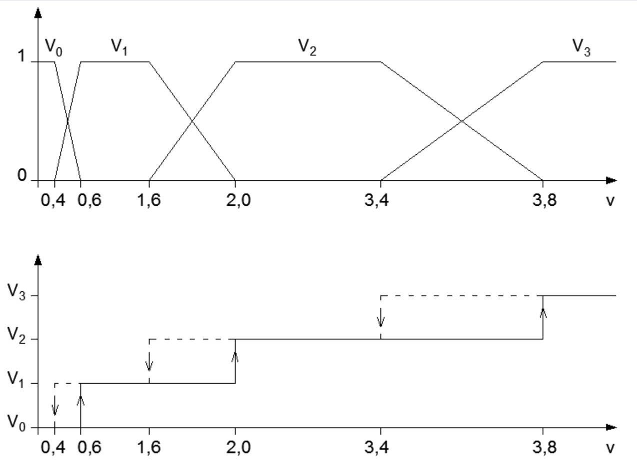
Qualitativness



Composition table

A_iA_j \ A_jA_k	o_rd_0	o_rd_u	o_td_u
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o_rd_s	o_rd_s	$o_r(d_s + d_u) = o_rd_{max\{s,u\}}$

Composition table

Table : All possible cases of adding qualitative distances for $d_s + d_u = d_{max\{s,u\}}$.

$+$	d_0	d_1	d_2	d_3
d_0	d_0	d_1	d_2	d_3
d_1	d_1	$\mathbf{d_1} d_2$	$\mathbf{d_2} d_3$	d_3
d_2	d_2	$\mathbf{d_2} d_3$	$\mathbf{d_2} d_3$	d_3
d_3	d_3	d_3	d_3	d_3

Velocity control

d \ dv	v_{-3}	v_{-2}	v_{-1}	v_0	v_1	v_2	v_3
d_0	Man_x^0	Man_x^0	$\boldsymbol{Dec_x^0}$	$\boldsymbol{Dec_x^0}$	$\boldsymbol{Dec_x^0}$	$\boldsymbol{Dec_x^0}$	$\boldsymbol{Dec_x^0}$
d_1	$\boldsymbol{Inc_x^0}$	Man_x^0	Man_x^0	$\boldsymbol{Dec_x^0}$	$\boldsymbol{Dec_x^0}$	$\boldsymbol{Dec_x^0}$	$\boldsymbol{Dec_x^0}$
d_2	$\boldsymbol{Inc_x^0}$	$\boldsymbol{Inc_x^0}$	Man_x^0	Man_x^0	Man_x^0	$\boldsymbol{Dec_x^0}$	$\boldsymbol{Dec_x^0}$
d_3	$\boldsymbol{Inc_x^0}$	$\boldsymbol{Inc_x^0}$	$\boldsymbol{Inc_x^0}$	$\boldsymbol{Inc_x^0}$	$\boldsymbol{Inc_x^0}$	$\boldsymbol{Inc_x^0}$	$\boldsymbol{Inc_x^0}$

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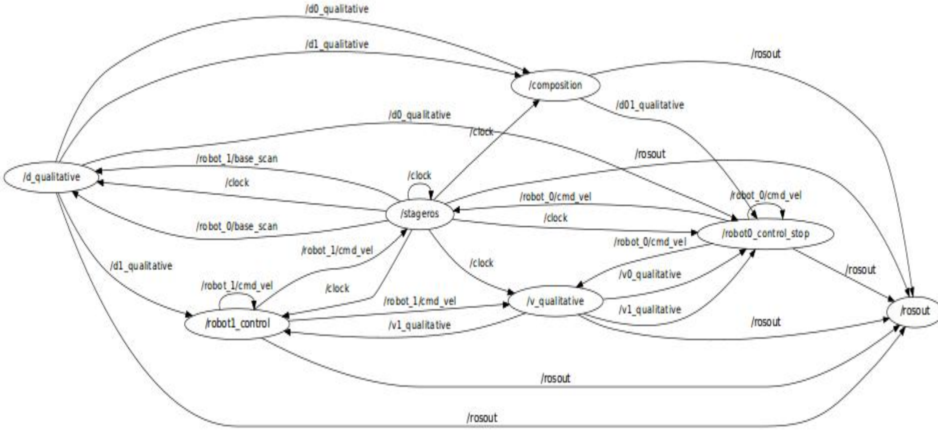

Simulation

Following

Simulation

Crash avoidance

Simulation

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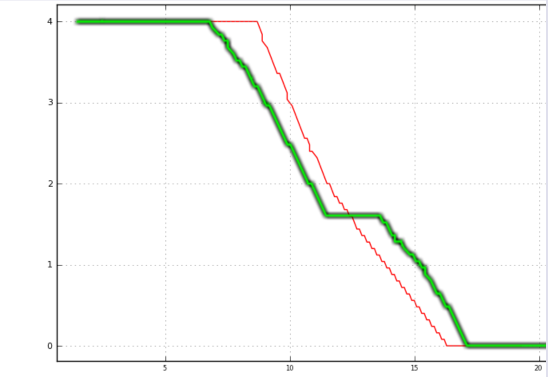

Figure : The A_i velocity change in the performed tests.

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Our application:

- solves the collision avoidance problem,
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Thank you for your attention

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